Assignment 27 — Parallel Resonance

OBJECTIVE

To investigate the parallel connection of R, L, and C.

EQUIPMENT REQUIRED

Qty	Apparatus
1	Electricity & Electronics Constructor EEC470
1	Basic Electricity and Electronics Kit EEC471-2
1	2–channel oscilloscope
1	Function generator 100 Hz–1 kHz, 20 V pk–pk sine *(eg, Feedback FG601)*

PREREQUISITE ASSIGNMENTS

Assignments 24 and 26

KNOWLEDGE LEVEL

See prerequisite assignments.

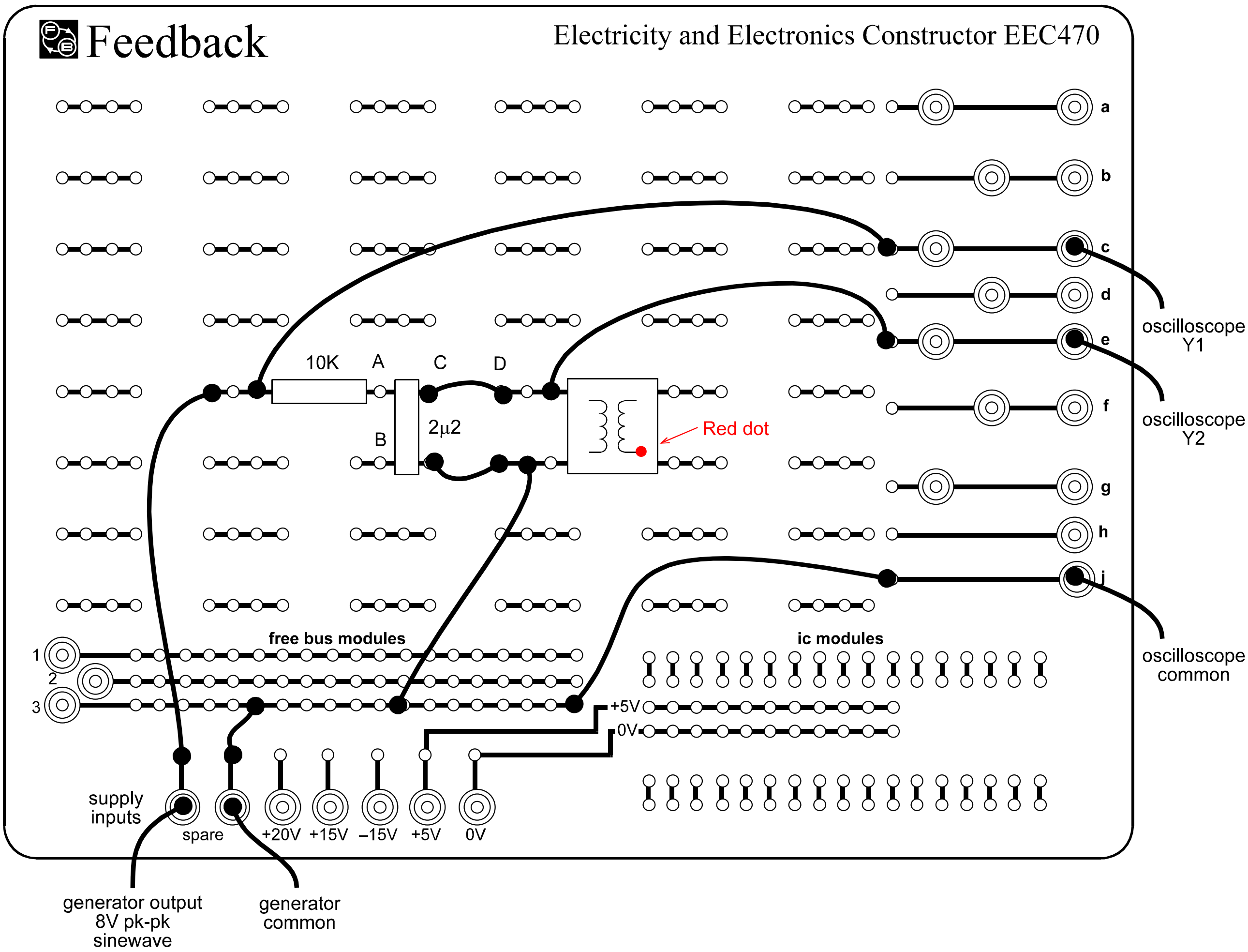

Feedback
Electricity and Electronics Constructor EEC470
10K
A
C
D
B
2μ2
Red dot
a
b
c
d
e
f
g
h
j
oscilloscope Y1
oscilloscope Y2
oscilloscope common
free bus modules
ic modules
1
2
3
+5V
0V
supply inputs
spare
+20V
+15V
–15V
+5V
0V
generator output 8V pk-pk sinewave
generator common

Fig 27.3

EXPERIMENTAL PROCEDURE

The circuit to be investigated is shown in fig 27.1. However, we will start with the simpler case where R is omitted.

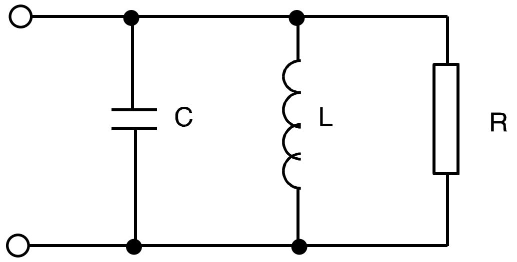


Fig 27.1

Connect up the circuit as shown in the patching diagram of fig 27.3 corresponding to the circuit diagram of fig 27.2.

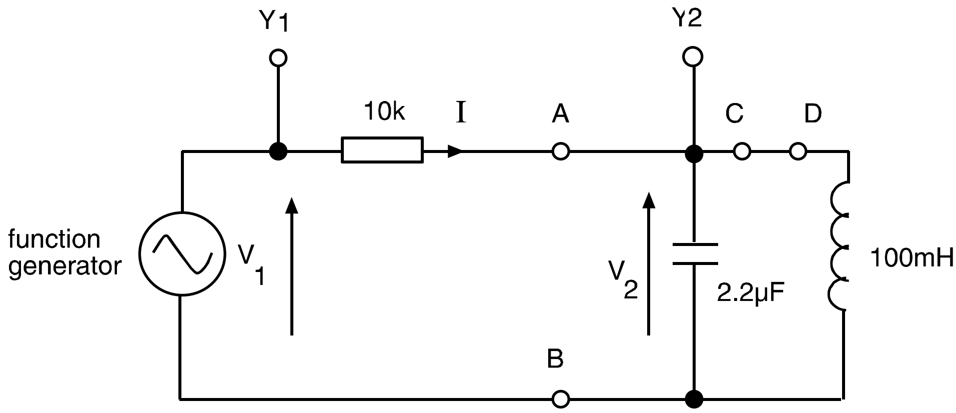


Fig 27.2

Set the function generator to give an output, V_1, of 6 V pk–pk at 100 Hz..

Set the oscilloscope to:

Y1 channel (V1) to 0.1 V/cm

Y2 channel (V2) to 200 mV/cm

Timebase to 1 ms/cm

Vary the frequency of the generator slowly from 100 Hz to 1 kHz, and notice the variation of the two voltages shown on the oscilloscope.

Questions

1. ***Does the voltage output of the generator vary appreciably?***

2. ***Does the voltage across the capacitor and inductor vary?***

3. ***What is the relationship between V_1, V_2 and I (fig 27.2)?***

4. ***Does I vary with frequency?***

5. ***Is V_2 a maximum or minimum at the resonant frequency of the circuit?***

6. ***Is I a maximum or a minimum at f_o?***

Set the generator to the frequency that gives a minimum in I (maximum V_2) and measure the frequency.

7. ***What is the resonant frequency?***

Calculate the current I at resonance from

$$I = \frac{V_1 - V_2}{10k\Omega}$$

Since the oscilloscope is being used for measurements, it will be found convenient to work in peak-to-peak values of voltage and current throughout. The values of impedance calculated will be the same as if rms values were used, provided that the same kind of measure is used for both voltage and current.

Questions

8. ***What is the impedance of the parallel LC circuit at resonance? (Find this from $\frac{V_2}{I}$)***

9. ***Is the impedance high or low at resonance?***

10. ***How does this compare with the series resonant circuit?***

Set the generator frequency to 150 Hz and the output amplitude to give 8 V pk–pk . If the dial accuracy of the generator used is not thought sufficient, a digital frequency meter may be used for greater accuracy.

Measure the voltage across the parallel LC circuit, V_2.

Copy the results table as shown in fig 27.4, reproduced at the end of this assignment, and tabulate your results.

Increase the generator frequency to 200 Hz, and reset the output amplitude to 8 V pk–pk.

Measure and record the resulting V_2.

Repeat this procedure for frequencies of, 250, 300, 350, 400, 450, 500, 550, 600, 700, 800, 900 and 1000 Hz.

Ensure that the amplitude of V_1 remains constant for each frequency setting.

Find the resonant frequency again and take a set of readings at f_o.

Calculate I and Z for each step, and enter your results in the appropriate spaces.

On a sheet of single-cycle logarithmic graph paper, draw a curves of Z against frequency, using the axes shown in fig 27.5.

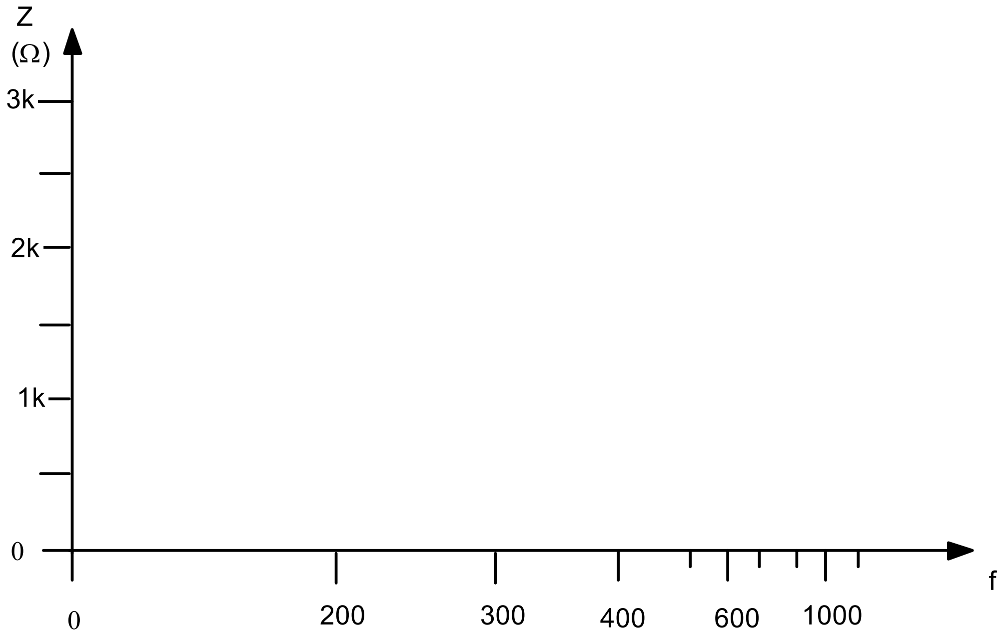


Fig 27.5

Now connect a 1 kΩ resistor between points A and B, in parallel with the resonant circuit, as shown in fig 27.6.

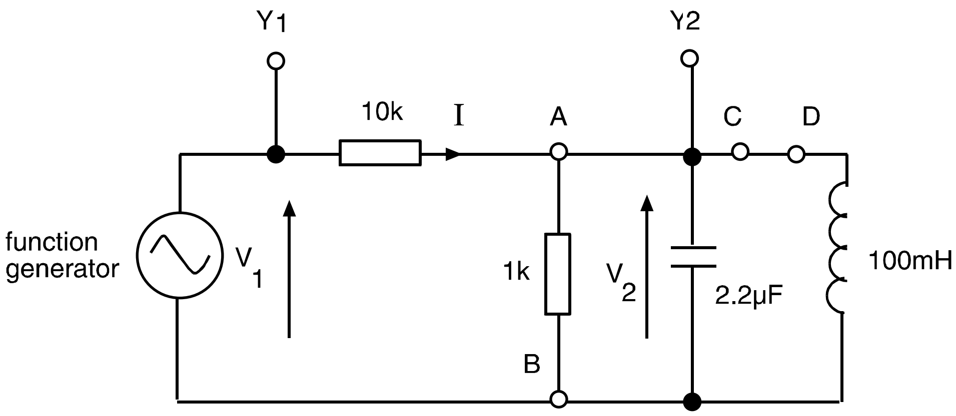


Fig 27.6

Set the generator frequency back to 150 Hz and the output to give 8 V pk–pk as before, and measure and record the resultant V_2.

Repeat the procedure for the same frequency steps as before, and draw the impedance curve on the same piece of graph paper as used for the previous part of the assignment.

Notice the different shapes of the two impedance curves.

Questions

11. Which circuit has the higher Q?

12. What is the resonant frequency of the circuit with R = 1 kΩ inserted?

13. Does this differ from when R was excluded?

Determine, from your curves, the bandwidths of the two circuits.

Using the expression relating Q with bandwidths and f_o, determine the Q values of the two circuits.

Let us analyse the circuit further:

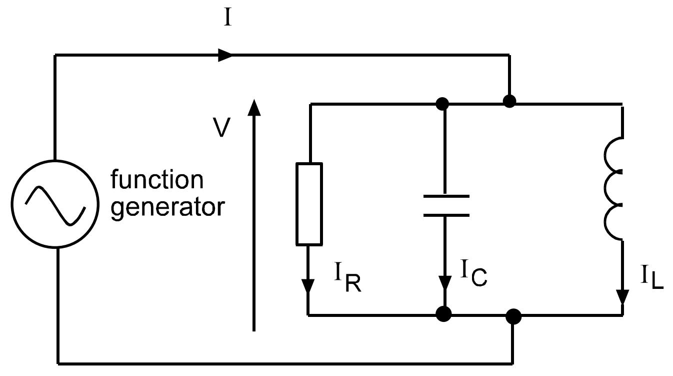


Fig 27.7

Referring to fig 27.7, at resonance the inductive reactance is numerically equal to the capacitive reactance, thus I_C and I_L will be equal and opposite. They will cancel each other out, as shown in the phasor diagram of fig 27.8.

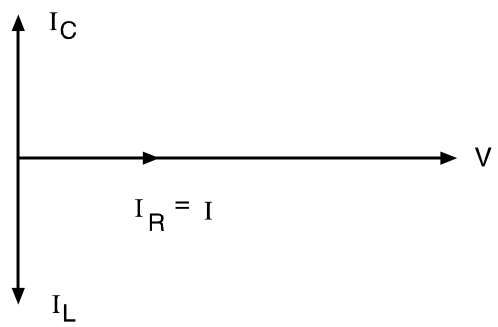


Fig 27.8

The resultant current at resonance is therefore only that required to supply the resistance, R.

Question

14. What was the measured value of this current?

Referring to fig 27.7 at resonance

$|I_C| = |I_L|$ thus $I_R = I$ at f_o.

For a parallel circuit we define the Magnification Factor Q as the ratio of reactive current to resistive current, at resonance:

ie, $$Q = \left|\frac{I_L}{I}\right| = \left|\frac{I_C}{I}\right|$$

Now $$|I_L| = \frac{V}{X_L} \text{ and } |I| = \frac{V}{R}$$

$$\therefore \left|\frac{I_L}{I}\right| = \frac{V}{X_L} \cdot \frac{R}{V} = \frac{R}{X_L}$$

$$\therefore \quad Q = \frac{R}{\omega_o L}$$

Similarly:

$$Q = \omega_o CR$$

Compare these expressions with those found for Q in the series resonant circuit.

Question

15. What relationship have they?

Remember, in the series case the R is the series resistance in the circuit, in the parallel case the R is the parallel resistance.

For clarity we may denote these by R_s and R_p for series and parallel resistance respectively.

The equations then become:

Series circuit: $$Q = \frac{\omega_o L}{R_s} = \frac{1}{\omega_o CR_s}$$

Parallel circuit: $$Q = \frac{R_p}{\omega_o L} = \omega_o CR_p$$

Using the expressions just found, calculate the Q value for the component values used.

Questions

16. How does this calculated value compare with the Q value found from the bandwidth?

17. What was the impedance of the RLC parallel circuit at resonance?

The formula for the impedance of a parallel RLC circuit is given by:

$$\frac{1}{Z} = \sqrt{\frac{1}{R_p{}^2} + \frac{1}{X^2}} \text{ (from Assignment 24)}$$

But at resonance the reactive terms cancel each other out, and the total resultant reactance is zero.

$$\therefore \text{at } f_o \qquad \frac{1}{Z} = \sqrt{\frac{1}{R_p{}^2}} = \frac{1}{R_p}$$

$Z = R_p$

Questions

18. Do your experimental findings agree with this?

In the case when there is no parallel resistance connected across the capacitor and inductor then $R_p = \infty$ (infinity).

19. What would the theoretical value of the impedance at resonance be?

20. Is it this in practice?

21. What would the theoretical value of Q be when $R_p = \infty$

22. Is it this in practice ?

There must therefore be some resistance present to limit Q and Z.

23. What other parallel resistances or impedances are there present across the LC circuit that might limit Q and Z?

An equivalent circuit may be drawn showing all the parallel resistances. This is shown in fig 27.9.

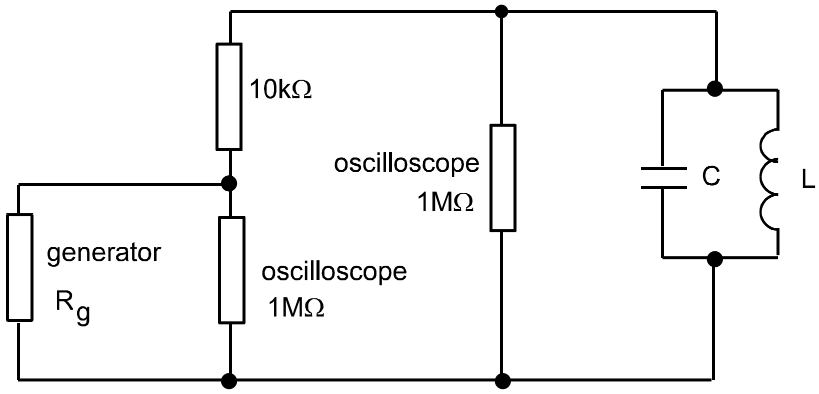


Fig 27.9

All these parallel paths cause a reduction in Q and in the impedance at resonance.

Question ***24.. What other component has resistance in the circuit?***

An equivalent circuit including the internal resistance of the inductor may be represented as in fig 27.10.

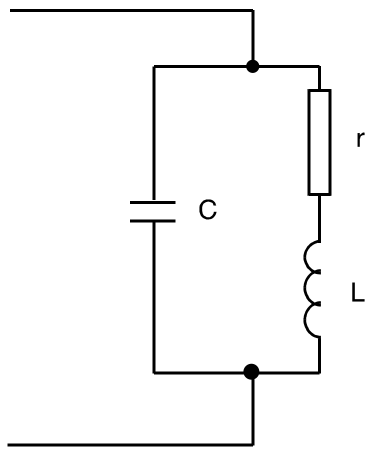


Fig 27.10

r represents the internal resistance of the inductor.

Let us investigate the effect of the internal resistance of the inductor on the Q and impedance by exaggerating the value of r by adding series resistance.

Remove the 1 kΩ resistor and connect a 100 Ω resistor between points C and D as shown in fig 27.11.

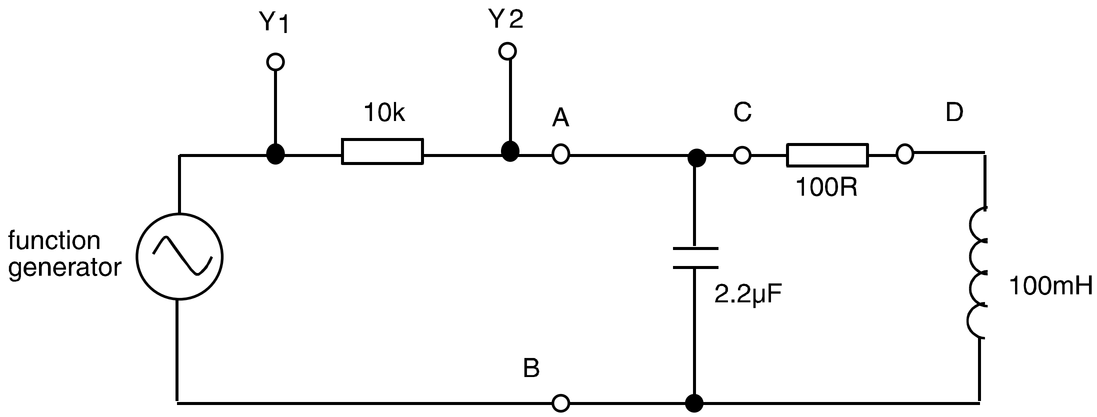


Fig 27.11

As previously, take readings of voltage for the same frequencies between 150 Hz and 1 kHz, and record them in another copy of the results table as in fig 27.4.

Draw the impedance curve on the same sheet of graph paper as before.

Questions

25. How does the Q of the circuit just tested (fig 27.11) compare with those of the other circuits?

26. What would you say must be done if a high Q circuit is to be achieved?

Find the resonant frequency of the circuit.

27. Is this the same as for the other two circuits?

Thus, not only does an appreciable internal resistance in the inductor drastically reduce the Q of a parallel tuned circuit, but also it changes the resonant frequency of the circuit as well.

The reason is shown below.

The currents in each branch are shown in fig 27.12.

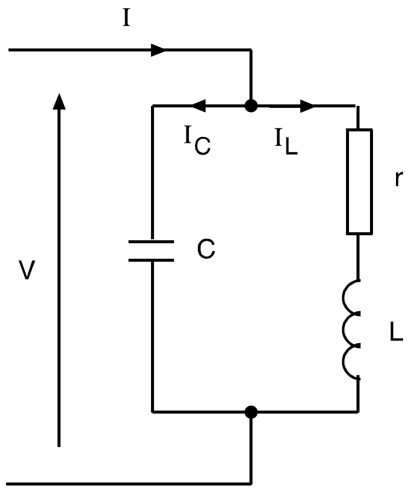


Fig 27.12

Now, the inductive branch is no longer a pure inductance, but an inductance and resistance in series. Thus the current I_L will no longer be equal and opposite to I_C. There will be a phase shift due to the presence of r. (See Assignment 20). The current through the inductor will lag the applied voltage by less than 90°. This is shown in fig 27.13.

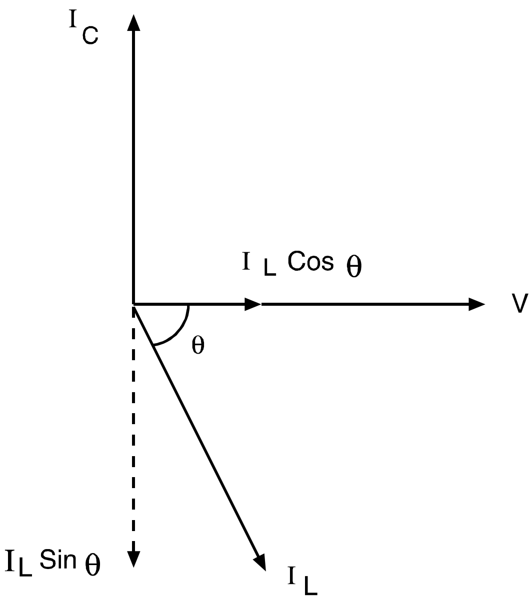


Fig 27.13

The inductor current, I_L, may be resolved into two components.

$|I_L|\sin\theta$, equal and opposite to I_C, and $|I_L|\cos\theta$, the current taken from the supply at resonance.

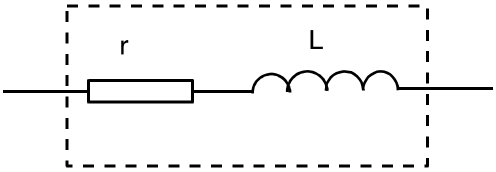
Thus:

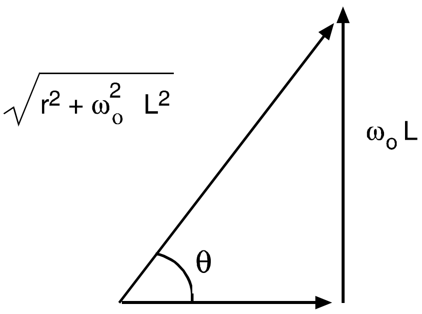
$$|I_C| = \frac{|V|}{X_C} = \frac{|V|}{\frac{1}{\omega_C}} = |V|\,\omega_O C \text{ at resonance}$$

$$\text{and}\quad |I_L| = \frac{|V|}{\sqrt{r^2+(\omega_o L)^2}} = \frac{|V|}{\sqrt{r^2+\omega_o{}^2L^2}} \text{ at resonance}$$

$$\therefore \quad |V|\omega_O C = \frac{|V|}{\sqrt{r^2+\omega_o{}^2L^2}}\sin\theta \qquad (1)$$

Let us consider the inductor, as in fig 27.14.





r

Fig 27.14

Now from the phasor diagram

$$\sin\theta = \frac{\omega_o L}{\sqrt{r^2+\omega_o{}^2L^2}}$$

therefore, substituting this in equation (1)

$$|V|w_oC = \frac{|V|}{\sqrt{r^2 + \omega_o{}^2L^2}} \times \frac{\omega_oL}{\sqrt{r^2 + \omega_o{}^2L^2}}$$

$$\therefore \quad C = \frac{L}{r^2 + \omega_o{}^2L^2}$$

$$\therefore \quad r^2 + \omega_o{}^2L^2 = \frac{L}{C}$$

$$\therefore \quad \omega_o{}^2L^2 = \frac{L}{C} - r^2$$

$$\therefore \quad \omega_o{}^2 = \frac{1}{LC} - \frac{r^2}{L^2}$$

$$\therefore \quad \omega_o = \sqrt{\frac{1}{LC} - \frac{r^2}{L^2}}$$

$$\therefore \quad f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{r^2}{L^2}}$$

Also we know $Z_o = \frac{V_o}{I_o}$ at resonance

At resonance, the current $I_0 = I_L \cos\theta$ (fig 27.13) and from fig 27.14:

$$\cos\theta = \frac{r}{\sqrt{r^2 + \omega_o{}^2L^2}}$$

$$\therefore \quad |I_L|\cos\theta = \frac{|V|}{\sqrt{r^2 + \omega_o{}^2L^2}} \times \frac{r}{\sqrt{r^2 + \omega_o{}^2L^2}}$$

$$= \frac{|V|r}{r^2 + \omega_o{}^2L^2}$$

$$\therefore \qquad |Z_o| = \frac{|V|}{\dfrac{|V|r}{r^2 + \omega_o^2 L^2}}$$

$$= \frac{r^2 + \omega_o^2 L^2}{r}$$

Substituting for ω_o^2

$$|Z_o| = \frac{r^2 + \left(\dfrac{1}{LC} - \dfrac{r^2}{L^2}\right)L^2}{r}$$

$$= r + \frac{L}{Cr} - r$$

$$\therefore \qquad Z_o = \frac{L}{Cr}$$

This is the value of the impedance at resonance and since the current and the voltage are in phase at resonance the impedance appears as a pure resistance.

It is often called the ***Dynamic Resistance*** of the circuit.

PRACTICAL CONSIDERATIONS AND APPLICATIONS

The points concerning losses in a resonant circuit which were enumerated in the applications notes of Assignment 26 apply equally for parallel tuned circuits as for series circuits. However the equivalent circuit becomes as in fig 27.15.

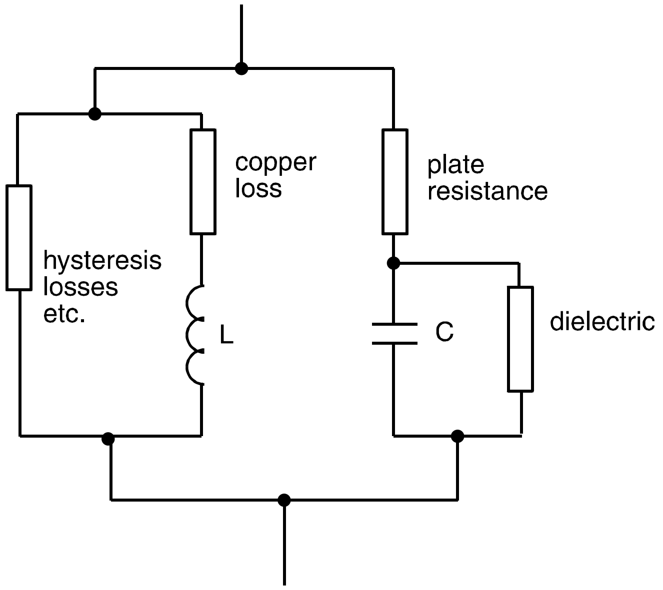


Fig 27.15

Again, these are normally represented by a single parallel loss resistance, R_p.

The difference in resonant frequency caused by the taking into account the series resistance of the coil is normally very small. This is because:

$$\omega_o L = Qr$$

$$\therefore \quad \frac{r}{L} = \frac{\omega_o}{Q}$$

$$\text{But} \quad \omega_o^2 = \frac{1}{LC} - \frac{r^2}{L^2}$$

$$\therefore \quad \omega_o^2 = \frac{1}{LC} - \frac{\omega_o^2}{Q^2}$$

$$\therefore \quad \omega_o^2 \left(1 - \frac{1}{Q^2}\right) = \frac{1}{LC}$$

Normally Q is in the region 10 to 200. With a typical value of $Q = 50$ the term $\frac{1}{Q^2}$ makes the equation

$$\omega_o^2 \approx \frac{1}{LC}$$

inaccurate by only one part in 2500,

$$\text{so} \quad f_o \approx \frac{1}{2\pi\sqrt{LC}}$$

with an accuracy in this case of one part in 5000.

This is the same equation as used in the series resonance case, and in practice provides a very close approximation to the true f_o.

The impedance of a parallel tuned circuit is at its highest at resonance. Thus the circuit is sometimes referred to as a 'rejecter circuit'. The parallel resonant circuit is perhaps more often used than the series circuit, and it is commonly used to provide a frequency dependent load across which a high voltage will be present at resonance, but a low voltage at frequencies away from resonance. This is shown in fig 27.16.

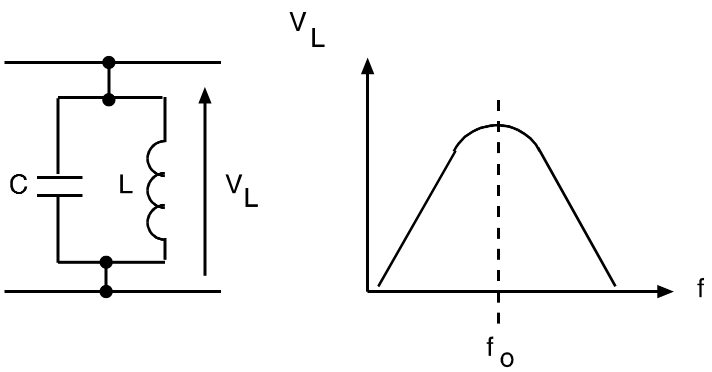


Fig 27.16

The parallel tuned circuit is used extensively in radio receivers and transmitters, and many other types of equipment.

RESULTS FOR ASSIGNMENT 27

$$i = \frac{V_1 - V_2}{10 \times 10^3}$$

$$= \frac{6 - 1.1}{10 \times 10^3}$$

$\therefore$ i = 0.49 mA pk–pk

frequency (Hz)	V1 (Vp–p)	V2 (Vp–p)	V1–V2 (Vp–p)	I (mA p–p)	Z (Ω)
150	8	0.115	7.885	0.789	145
200	8	0.210	7.790	0.779	269
250	8	0.560	7.440	0.744	759
300	8	1.160	6.840	0.684	170
350	8	0.570	7.430	0.743	760
400	8	0.360	7.640	0.764	474
450	8	0.250	7.750	0.775	321
500	8	0.190	7.810	0.781	244
550	8	0.160	7.840	0.784	205
600	8	0.140	7.860	0.786	179
700	8	0.100	7.900	0.790	127
800	8	0.084	7.916	0.792	106
900	8	0.074	7.926	0.793	94
1000	8	0.064	7.936	0.794	81
f_0 = 275	8	1.280	6.720	0.672	1904

Fig 27.4

Parallel LC circuit

frequency (Hz)	V1 (Vp–p)	V2 (Vp–p)	V1–V2 (Vp–p)	I (mA p–p)	Z (Ω)
150	8	0.102	7.898	0.790	129
200	8	0.180	7.820	0.782	231
250	8	0.335	7.665	0.767	435
300	8	0.550	7.450	0.745	733
350	8	0.460	7.540	0.754	613
400	8	0.325	7.675	0.768	422
450	8	0.235	7.765	0.777	301
500	8	0.190	7.810	0.781	244
550	8	0.155	7.845	0.785	199
600	8	0.132	7.868	0.787	167
700	8	0.102	7.898	0.790	129
800	8	0.084	7.916	0.792	106
900	8	0.074	7.926	0.793	94
1000	8	0.064	7.936	0.794	81
f0 = 310	8	0.560	7.440	0.744	757

Fig 27.4

Parallel RLC circuit

For a parallel circuit the bandwidth is defined as the difference between two frequencies f_1 and f_2 at which the impedance has fallen to 0.707 x impedance at f_0.

From the graphical results:

	without R	with R = 1 kΩ
	f_0 =275 Hz	f_0 = 310 Hz
	Z_0 = 1904 Ω	Z_0 = 757 Ω
	0.707 x Z_0 = 1346 Ω	0.707 x Z_0 = 535 Ω
	∴ f_1 = 265 Hz	∴ f_1 = 270 Hz
	f_2 = 305 Hz	f_2 = 360 Hz
	∴ BW = 40 Hz	∴ BW = 90 Hz

From the formula $Q = \frac{f_O}{BW}$

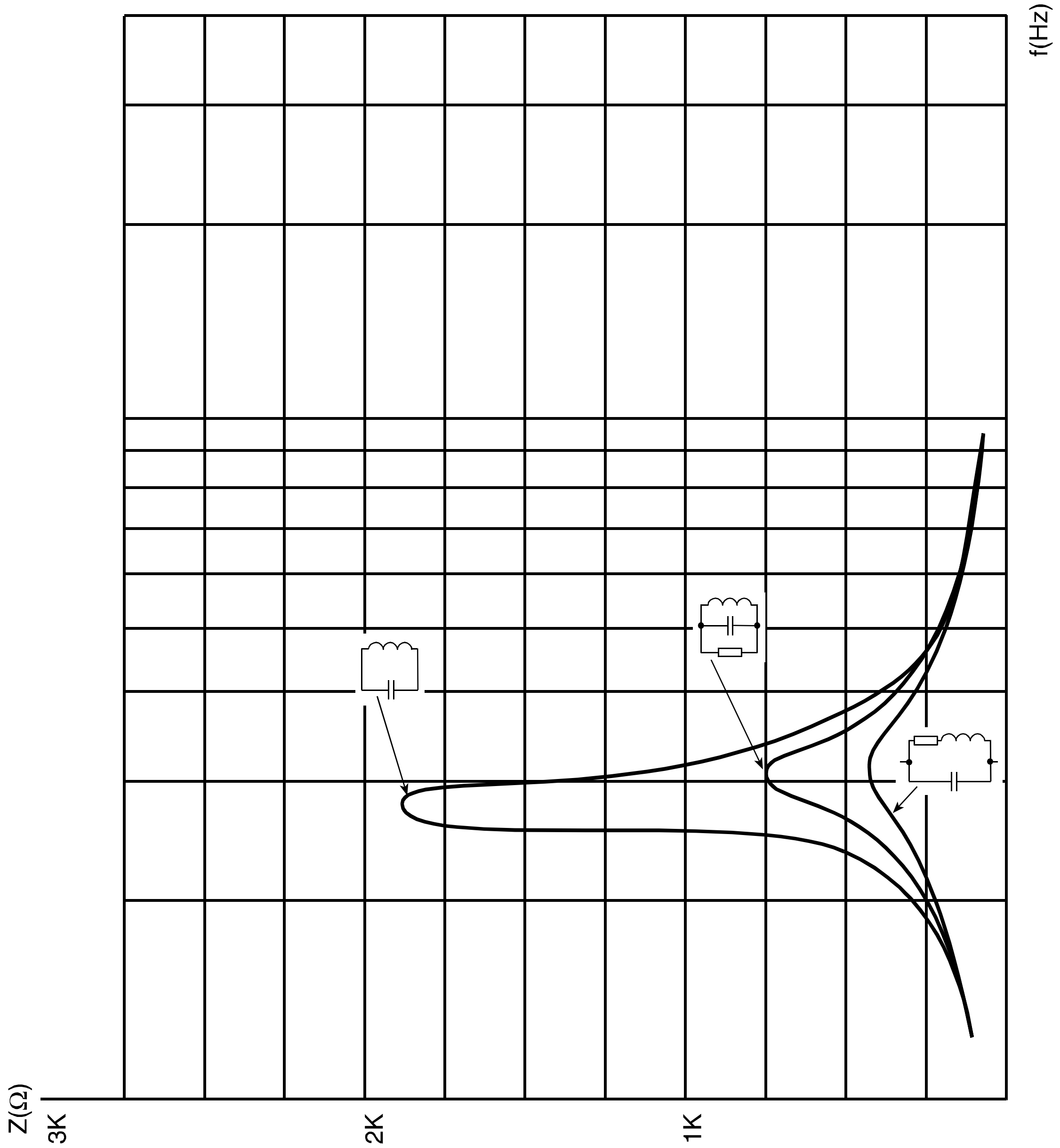
without R $Q = \frac{275}{40}$ with R = 1 kΩ $Q = \frac{310}{90}$

∴ Q = 6.8 ∴ Q = 3.4

frequency (Hz)	V1 (Vp–p)	V2 (Vp–p)	V1–V2 (Vp–p)	I (mA p–p)	Z (Ω)
150	8	0.100	7.900	0.790	127
200	8	0.165	7.835	0.784	210
250	8	0.260	7.740	0.774	336
300	8	0.335	7.665	0.777	431
350	8	0.300	7.700	0.770	390
400	8	0.250	7.750	0.775	323
450	8	0.200	7.800	0.780	256
500	8	0.170	7.830	0.783	217
550	8	0.140	7.860	0.786	178
600	8	0.120	7.880	0.788	152
700	8	0.096	7.904	0.790	122
800	8	0.080	7.920	0.792	101
900	8	0.070	7.930	0.793	88
1000	8	0.060	7.940	0.794	76
f0 = 315	8	0.340	7.660	0.766	444

Fig 27.4

Parallel LC circuit with increased inductor resistance



Results graph for Fig 27.4 (three parallel circuits)

ANSWERS TO ASSIGNMENT 27

1. No

2. Yes

3. $i = \frac{V_1 - V_2}{10k\Omega}$ A

4. Yes, because V_1 and the resistance are constant while V_2 varies with frequency.

5. V_2 is a maximum at f_0.

6. i is a minimum at f_0.

7. f_0 = 277 Hz

8. For the parallel circuit:

   $Z = \frac{V_2}{i}$

   $= \frac{1.1}{0.49 \times 10^{-3}}$

   $\therefore Z = 2245\ \Omega$

9. Z is high at f_0

10. In a series RLC circuit Z is low at f_0.

11. The circuit without the resistor in parallel has the higher Q.

12. f_0 = 310 Hz

13. Yes

14. 0.74 mA

15. The expressions for Q in terms of w_0, R, L and C for a parallel circuit are the inverse of those for a series circuit.

16. From the given value of the components:

    $Q = \frac{R}{\omega_0 L}$ or $Q = \omega_0 C R_p$

$= \dfrac{10^3}{2\pi \times 310 \times 100 \times 10^3}$ $\qquad = 2\pi \text{ x } 310 \text{ x } 2.2 \text{ x } 10^{-6} \text{ x } 10^3$

$\therefore$ Q = 5.14 $\qquad \therefore$ Q = 4.28

The difference in value is due to the actual resonant frequency, 310 Hz, used in the calculation not being the same as that which may be calculated using the given component values.

In comparison, from the bandwidth Q = 3.4

17. 757 Ω

18. At f_0: From the experimental results R_p =Z = 757 Ω

    From the given component value R_p = 1 kΩ

19. Infinity

20. No

21. Infinity

22. No

23. The following impedances are effectively across the parallel LC circuit:

    Input impedance of oscilloscope Y amplifiers.

    Output impedance of function generator.

    10 kΩ current monitoring resistor.

24. The inductor has resistance.

25. The Q of the circuit with extra resistance in series with the inductor is lower than those of the other circuits.

26. To achieve a high Q circuit, the value of resistance must be as low as possible.

27. f_0 = 315 Hz for the circuit with increased inductor resistance.

    In comparison:

    f_0 = 275 Hz for the basic LC parallel circuit.

    f_0 = 310 Hz for the circuit with a 1 kΩ resistor in parallel with the LC circuit.

frequency (Hz)	V1 (Vp–p)	V2 (Vp–p)	V1–V2 (Vp–p)	I (mA p–p)	Z (Ω)
150					
200					
250					
300					
350					
400					
450					
500					
550					
600					
700					
800					
900					
1000					

Fig 27.4